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# Scheduling for client-driven capacitated systems

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# Scheduling for client-driven capacitated systems

by

Debra Sprunger Bishop

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
DOCTOR OF PHILOSOPHY

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# NOTATION

| | |
|---|---|
| CDCS | client-driven capacitated system |
| $P_k$ | project k |
| $T_{ik}$ | task $i$ within $P_k$ |
| $R_j$ | resource j |
| $SF$ | skill factor |
| $EF$ | effort factor |
| $IF$ | interruption factor |
| $U_R$ | proportion of average work resource $R_j$ will accomplish |
| $TR$ | total revenue |
| $TC$ | total cost |
| $PC$ | penalty cost |
| $M_t$ | profit for current period |
| $DC$ | delay cost per period |
| $TDC$ | total delay cost |
| $t$ | time period |
| $E_n$ | one-step transition matrix |

| | |
|---|---|
| $WR$ | waiting resource state |
| $OH$ | on-hold state |
| $C$ | completion state |
| $\alpha$ | expected on-hold duration each occurrence |
| $p_{ij}$ | probability of going from $i$ to $j$ at end of period $n$ |
| $a_n$ | probability of going from $WR$ to $WR$ at end of period $n$ |
| $b_n$ | probability of going from $WR$ to $OH$ at end of period $n$ |
| $c_n$ | probability of going from $WR$ to $C$ at end of period $n$ |
| $X_t$ | expected periods until completion |
| $W_t$ | expected periods to completion excluding on-hold time |
| $DD$ | due date |
| $D_t$ | expected number of periods until due date |
| $S$ | slack |
| $ED$ | expected number of periods delayed |
| $H_t$ | expected periods on hold |

# CHAPTER 1. INTRODUCTION

## Problem Rationale

Environments in which a particular knowledge or skill is used to transform a concept or idea into a final service or product at the direction of a client (either internal or external), have a unique combination of characteristics that distinguish them from many capacitated systems. Key differences distinguishing these systems from others in research (e.g., manufacturing systems) include the iterative completion nature and the frequent resource interruption factor. Particular elements of these systems include: limited, specialized resources, parallel processes, client prioritization, client-controlled completion deadlines, and delay penalties. We observe the resource-project relationships of these systems as shown in Figure 1.1 with $P$ representing projects, $T$ representing tasks within a project, and $R$ representing resources. We then can observe these environments in a wide variety of disciplines such as those given in Table 1.1.

Each of these categories shares numerous characteristics, including but not limited to the following:

- RESOURCE CHARACTERISTICS
    - limited, variable resource availability

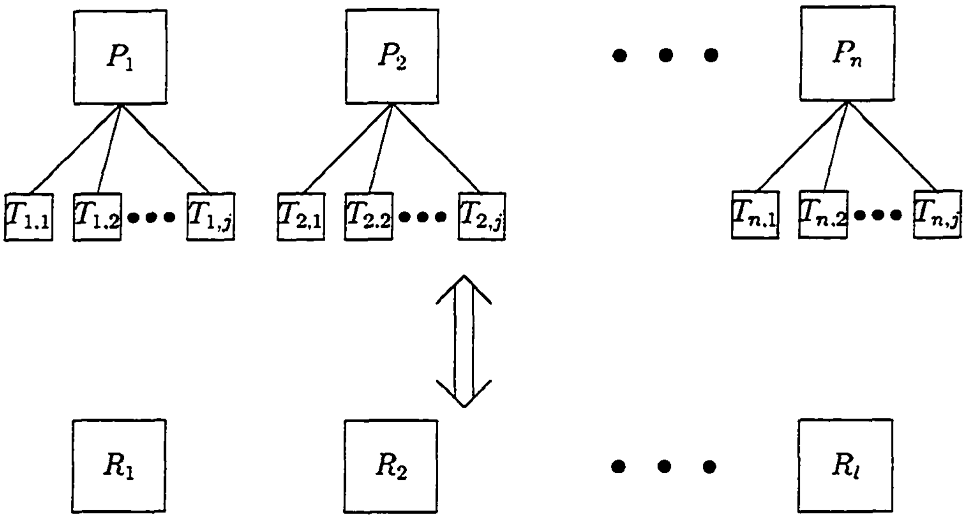

Figure 1.1: Project-Resource Relationship

  - variability in resource experience, capability, and effort level
  - after assignment a project "belongs" to a resource
  - variable interruption level among resources
- CLIENT CHARACTERISTICS
  - client prioritization
  - varying demand level among clients
- PROJECT CHARACTERISTICS
  - multiple projects in progress concurrently

Table 1.1: Table of Project Environments

| ENVIRONMENTS | |
|---|---|
| **CATEGORY** | **RESOURCE** |
| Building Layout | Architect |
| Subdivision Layout | Civil Engineer |
| Tax Return Preparation | Accountant |
| Financial Portfolio | Financial Consultant |
| Software Modification | Software Engineer |
| Book, Newspaper, or Journal Article | Writer or Journalist |
| Custom-made Clothing Design | Seamstress |

  - staggered, identifiable, start and completion dates for projects
  - multiple project categories within a particular firm
  - similar (but not duplicate) work content among projects within a given category
  - varying inter-project complexity and duration
  - history of similar projects to draw knowledge from
  - potentially iterative nature, task order and project content may be variable and partially unknown at initiation

- COMPLETION CHARACTERISTICS
  - time limit (due date deadline) for each project completion
  - review or accountability points throughout the project's life

  - projects can be temporarily set aside to reassign a resource onto another higher priority project
  - multiple feasible paths to successful/unsuccessful completion
  - certain decisions may result in dead-ends, reworks, redundancy, or undetected flaws

Each of the previously listed characteristic categories has components that must be coordinated or controlled in order to achieve two critical objectives for each project, namely,

- satisfactory quality at completion, and
- timeliness of completion.

## Research Overview

Given the role of client and the limited resource availability, we refer to such a system as a **Client-Driven Capacitated System (CDCS)**.

The objective of this research has been first, to identify and categorize common characteristics among CDCSs which seem to have been largely ignored by researchers concerned with productivity. Then, information from representative systems has been used to derive a quantitative, prescriptive model by which resource allocation decisions can be made. The basis for allocation decisions is such that for a given resource assignment period the profit generated by the CDCS can be maximized within a given set of parameters.

We begin with a discussion of related research, continue with the characterization of the CDCS, and then development of a non-stationary Markov model. We then develop resource allocation criteria, corresponding decision methodology, and present a representative example. We conclude with possible future extensions and applications.

# CHAPTER 2. LITERATURE REVIEW

Related research in the literature can be found in areas such as construction management. optimization methods. manufacturing processes. and Bayesian approaches.

## Scheduling Research Comparisons

Recent research in scheduling provides insight into and comparative analysis for resource allocation methods. Randhawa and Smith [1995] consider the assignment of jobs to processors that have unequal capabilities and tardiness penalties. in applications where there are sequence-dependent set-up times such as a paper plant assigning products to different paper machines. They state that "obtaining an optimum solution for all but the smallest systems is not practical in common. everyday systems." Guinet [1991] focused on a mean tardy time minimization linear programming approach in the area of textile manufacturing. but again this method was so computationally complex as to be intractable for all but the simplest problems. Randhawa and Smith [1995] state that. "an understanding of how relationships between the parallel processors. scheduling systems and product and job distributions affect system performance may lead to decision rules that can give a feasible. satisfactory schedule. Although the schedule may not be optimum, the tradeoffs in programming and computation time to allow for more frequent. and hence more accurate. schedules will be beneficial." Hence a decision-rule based model seems appropriate. They also note that though significant research has been done in scheduling, little scheduling research has occurred directly related to their application.

Johnson [1974] shows time series model applications and this approach was also considered due to the time component of the CDCS model, but once again this does not seem entirely appropriate due to the nature of such systems.

Research on Make-to-Order (MTO) parts has interesting parallels because of the parallel scheduling of activities and client-driven design aspects. Handfield [1994] proposed a concurrent engineering approach for MTO parts based on incremental improvements (as opposed to breakthrough products). He looked at six hypotheses and a sample of 31 MTO products. Then using an analysis of variance approach he tested his hypotheses. The results of his research suggested that by sharing imperfect information at various points, subsequent activities could be started before previous ones were complete and reduce total development time, even in a presumably sequence-dependent environment. The aspects that distinguish MTO systems from CDCSs are the production phase and sequence-dependent activities typified by a manufacturing environment, but the concept of sharing imperfect information is still useful. Tuttle [1994] also looks at resource allocation models, but in reference to large-scale, one-time projects such as home construction.

In an overview of neural network application advances for manufacturing, Zhang and Huang [1995] discuss the application of neural networks for scheduling because of the ability to adapt to sudden, unforeseen changes. They noted, however, limitations in applicability due to today's computer technology.

During the implementation phase of most large-scale one-time projects the order and duration of tasks are typically well-defined (e.g., construction), but with the CDCSs outlined in the introduction the situation is less explicit. With a limited number of resources working on several different projects at once, resource allocation decisions are critical for each assignment period. In the typical project implementation as shown in previous research [Al-Bahar 1990, Arditi 1989, Laufer 1990], the task order may be initially well-defined. However, unanticipated delays or priority changes may occur due to circumstances such as: changes in the scope of work or details, inadequate information, or conflicting objectives that must be considered. These all contribute to making conventional tools such as PERT or CPM too simplistic to fully address the critical components of many types of projects, including CDCSs.

Also, Schei [1990] points out that projects that are of relatively short duration and limited scope make proper scheduling of activities from the onset essential. Also, the variability of content and duration, and staggered project intiation dates, as well as the varying resource capabilities required at different project points, all contribute to model difficulty. He emphasizes, though, that with many projects running concurrently, proper resource allocation is critical.

Scheduling is well-researched, but as noted by Randhawa and Smith [1995] there are areas and applications where little emphasis has been placed. Though there are numerous related topics from which parallels can be identified, a method that considers resource allocation for client-driven capacitated systems (CDCSs) as discussed in the introduction appears to be an area overlooked in the field of scheduling models.

## Bayesian Approach

In addition to the field of scheduling research. another area in which many researchers have concentrated efforts is Bayesian theory. Bayesian decision analysis provides a means for identifying the best decision based on predetermined objectives in a setting where uncertainty exists (i.e.. stochastic in nature) [Muth 1963. Smith 1988]. It is particularly useful when there is both uncertainty about task duration and some historical criteria upon which to base the decision [Smith 1988].

Cooper [1993] drew the conclusion that for one-time construction projects it was possible to learn and improve performance on subsequent projects. This concept could be useful for a CDCS. He states the inadequacy of CPM and similar tools due to the iterative nature inherent in a CDCS. He used simulation. but it appears a Bayesian approach [Smith 1988] could be most beneficial.

Martin [1975] further illustrates the applicability of a Bayesian decision approach in conjunction with models using Markov chains with uncertain transition probabilities. particularly sequential decision models.

Research indicates that the Bayesian approach is useful for assigning initial transition probabilities and initial resource characteristics using a historical basis of knowledge. This information can then be used for project duration estimation. As additional information is obtained through actual activity occurrences. the original probability distribution can then be altered to reflect this new information using Bayes' Theorem [Morgan 1966].

Therefore. in applications where inter-project similarity exists and a historical reference is available (or can be established). application of Bayes' Theorem is useful in accounting for the probabilistic nature of the data.

# CHAPTER 3. CHARACTERIZATION OF A CDCS

## Observations on a CDCS

By observing the processes in representative samples of CDCSs. fundamental characteristics for a typical CDCS were determined. First. a set of interview questions were developed for this study and are given in Appendix A. Three representative CDCSs were chosen. namely. an insurance agency [Webb 1995]. a private accounting firm [DeHamer 1995]. and a civil engineering firm [Bishop 1995]. One representative of each CDCS was interviewed. The interview was taped and transcribed at a later time. Following completion of all three interviews, the gathered information was examined to obtain general observations on the processes. From these observations. generic characteristics of the three systems were categorized. See Appendix B for details. From these generic characteristics. key parameters were identified as important for consideration in the model.

To illustrate. an example scenario could include a company $X$ that has three on-going projects. $P_1$. $P_2$. and $P_3$ that are in varying states of completion. having total expected revenue of \$8.000, \$10,000. and \$5,000. respectively. $P_1$ and $P_2$ have no penalty cost but $P_3$ is \$100/day. $P_1$ is due in fourteen days, $P_2$ in ten days. and $P_3$ has twenty-three days remaining. The company has two resources. The decision-maker is faced with the decision of how to allocate the three projects between the two resources to find the "best" fit for the current period in a manner that will maximize the profit for the current assignment period. The manner in which this decision can be made will be detailed in the remainder of the dissertation.

## Model Characteristics

As noted previously, four key characteristic categories were identified: resource, client, project, and completion. Within each category certain parameters, notation and assumptions have been identified. Given the assessment of the generic characteristics, the CDCS can be modeled as shown in the following sections.

### Project Characteristics

A CDCS has a set of ongoing projects

$$P = \{P_k, k = 1, \cdots, p\}$$

where $p$ is the total number of projects ready to be worked on at the start of a given assignment period and each $P_k$ is in one of the possible project states to be described later. Within each project, $P_k$, are certain tasks, $T_{ik}$.

For each project, at its inception, the total revenue, $TR_k$, upon completion is determined. This is agreed upon between the client and CDCS prior to the project initiation, but can be renegotiatied and changed during the course of the project if necessary due to project modification, external delays, etc. This potentially iterative nature is a key model characteristic that must be considered. The total revenue generated in relationship to the time and effort expenditures is inherently different for different project types; some by nature generate more revenue with less total resource cost required.

For all projects within a given CDCS, multiple, well-defined project categories exist unless the project constitutes a new area of endeavour. The number of project categories will be dependent on the particular CDCS. For some project categories, the work content can be easily subdivided into smaller component tasks while for others it is more difficult. Though each project is different from its predecessors and others currently ongoing, due to the limited project scope, there are similarities from which information can be gained. Therefore, within a category there will be task content similarities, though there will be inherent variablity in complexity and duration of the projects. This variability will be much more pronounced between categories, hence the basis for categorizing the projects as necessary to exploit the inherent similarities and differences.

### Completion Characteristics

For a given CDCS, a resource-project pairing is made at the start of each assignment period. The assignment period, $t$, is a preselected duration of time, typically a day, half-day, hour, etc. depending on the project and component task duration. The assignment period should be a reasonably small increment such that state changes can be assumed to be made at the end of the period and work can be partitioned into discrete periods when possible.

From the previous observations on CDCSs, the possible states for a project at the start of the assignment period were identified as follows.

WR: the project is in some state of partial completion and available and waiting for resources to be assigned. This includes new projects for which work has not yet been initiated and for which the transition probabilities are based entirely on historical data.

OH: on-hold for reasons beyond the control of the CDCS and cannot be worked on during the current assignment period.

C: assumed complete. meaning the expectations for completion have been met by the CDCS and the project is then removed from further resource assignment consideration.

It is assumed that for the duration of the assignment period the project will remain in the current state with any progress dependent on the resource allocation decision made at the start of the period. It is possible for a project to be put on-hold or reach completion during the assignment period, but this is considered to happen at the end of the assignment period. This is reasonable when we consider a project that is ready to be worked on at the start of a period would have a low probability of being put on-hold. It is also assumed that the transition to the next state is based only on the current state and not on previous state transitions (i.e, these systems have the Markov Property [Wolff 1989]).

A partial list of reasons for which projects enter the on-hold state include,

1. lack of client finances,
2. plan or concept modification,
3. weather delay,
4. or, waiting for information or consensus from some outside entity.

The on-hold state is considered to be externally (client or third-party) driven. When a project is on-hold it is simply withdrawn from consideration in the current period. In discussions with representative CDCSs concerning the frequency and duration of on-hold time, there tends to be random occurrences across project categories and over the life of a given project. The random behaviour should be reflected in the model.

Table 3.1: Historical Reference Information

| **Project Information** |
|---|
| project categories |
| transition probabilities |
| total project work content |
| project on-hold rate |
| length of each on-hold duration |
| reason for on-hold decision |
| total revenue generated |
| total delay cost |
| **Resource Information** |
| capability limitations |
| project assignments |
| number of interruptions per period |
| effort measure |
| error types and frequencies |

Historical reference information as shown in Table 3.1 can be available or accumulated for decision-making. This includes the transition probabilities to accessible states (including completion and on-hold). This information is useful for obtaining an expected initial completion duration, $X_1$, as well as establishing an agreed-upon due date, $DD$, prior to initiation.

Transition probabilities, the probability of going from one state to another, can vary between projects and be dynamic over the life of a project, e.g., the likelihood of transitioning to the completion state will likely increase as work is completed on the project. An initial probability is based on the expected value of a historical distribution from data recorded for previous projects of a similar nature. Once in progress, if greater or less work is accomplished than anticipated, or updated project characteristic information is received, the transition probabilities can be modified and refined over the project life using Bayes' Theorem [Smith 1988, Morgan 1966].

It is possible that due to limited resources a project may not be worked on even though it is available and requires attention. Intermittent work is possible, as a project can be pre-empted by a higher priority project. Some duplication of effort may be required upon returning to a pre-empted project (analogous to re-set-up time after changing products on a given machine) and should be reflected in the transition probabilities.

To account for the possibility of error and subsequent error correction time, the transition probability from $WR$ to $C$ can be altered at the end of the assignment period to reflect additional expected time for error correction.

Satisfactory completion is based on clearly defined, pre-established criteria. Project cancellation removes a project from consideration. Also, due to the types of projects in these CDCSs, there are multiple, unique resource allocation sequences to successful project completion.

### Resource Characteristics

At the start of each period there are a limited number of resources,

$$R = \{R_j, j = 1, \cdots, r\},$$

where $r$ is the total resources available on a given day.

The total number of resources available on a given day will vary due to human factor issues such as vacations. illnesses. terminations. etc.. and equipment issues such as breakdowns. maintenance. etc. Human resources are the main emphasis in CDCSs. hence an emphasis on human resource characteristics.

Associated with each human resource are certain distinguishing factors that should be incorporated into the model:

- skill factor. $SF$. based on level of expertise and amount of experience
- effort factor. $EF$. based on historical effort level on tasks. and
- interruption factor. $IF$. based on the historical total duration of interruptions per period the given resource encounters (e.g.. phone calls).

These can be combined to determine $U_{R_j}$, the fraction of the average work that can be accomplished by a given resource in a given time period. The three factors help distinguish the amount of work that can be expected to be accomplished by the given resource in relation to the overall average. For example. a resource with less expertise. less effort level and greater level of interruptions can be expected to accomplish significantly less than the average resource in a given time period. Also. each resource. $R_j$. has capability limitations: as only certain resources may be capable of doing certain tasks within a given project. Some resources will have more assignment flexibility than others. This is due to the nature of the CDCSs: expertise is required for different portions of each project and specific resource responsibilities exist. Because of this. there is a subset of the total resources that is feasible at different stages of a given project .

In addition, each human resource can have a distribution of average work accomplished for a given period based on historical data. This data can be dynamic and altered to reflect changes in any of the aforementioned resource factors using Bayes' Theorem [Morgan 1966, Smith 1988]. This data modification is a key component of the model.

This resource capability information is necessary to:

1. determine feasible assignments of resources to project tasks (at the start of an assignment period one resource is allocated to one project for the entire period), and
2. anticipate the expected amount of work that can be accomplished by this resource over the assignment period.

Associated with each human resource is a historical basis of error type and frequency. The type and level of errors is CDCS-dependent, but can be incorporated into the transition probabilities. Lastly, though not explicit in the model, a resource that finds itself idle during the course of an assignment period, can be reassigned to the next available project on the prioritized project list.

At the start of an assignment period, because of limited resources, a decision must be made concerning the best resource allocation given the current state of each of the projects. Due to limited resources, only a portion of the projects available and waiting resources can be assigned for a given assignment period, hence the term capacitated. For a given assignment period, all available resources will be assigned if work is available. Once assigned to a project, the resource remains with the project until either the project is finished, or the project requires capabilities not held by the current resource and another must take over. Hence, a resource will likely have multiple, ongoing projects among which they alternate.

### Client Characteristics

The model as formulated results in client prioritization based on varying revenue levels. Though not addressed, a model extension could be formulated in which clients are prioritized by alternative methods, including varying demand levels.

Inherent in a CDCS is the possibility of rejecting a potential client on the basis of a number of criteria, including failure to furnish revenues for previously completed projects. If the reason for rejection is due to overloaded resource commitments on current projects, the model has the capability of identifying this problem as well. If a potential project is rejected prior to initiation, it is simply reflected by lack of inclusion in the list of projects available for resource assignment.

Considering the four components of a CDCS and the iterative nature of projects, at the start of each period assignment decisions must be made. The primary factor affecting the decision is the selection of a suitable objective function. Possible objectives could be cost, time, profit, etc. The objective function selected is calculated from project and resource information and if chosen correctly will result in the "best" allocation of the limited, available resources for the current assignment period.

# CHAPTER 4. NON-STATIONARY MARKOV MODEL

We have the following CDCS project and resource information available at the beginning of each assignment period.

Project dependent information includes.

- expected transition probabilities useful for determining the number of periods (transitions) remaining until completion. $X_t$.
- likelihood of the project going on hold and for what reasons.
- likelihood of errors over the project life. the expected number and duration of errors. and
- a completion deadline commitment.

Resource dependent information includes,

- experience level.
- effort level.
- interruption level. and
- task capabilites.

This information. as detailed in the previous section. is utilized in the development of a non-stationary Markov model.

The state space for each project $P_k$ is:

WR: waiting for resources

OH: on-hold. and

C: assumed complete.

The corresponding non-stationary one-step transition probability matrix. $E$. for project $P_k$ is given in Table 4.1. The state-space diagram defining the possible transitions is shown in Figure 4.1.

Table 4.1: Transition Matrix for step $n$

$$E_{k,n} = \begin{array}{c|ccc|} WR & a_n & b_n & c_n \\ OH & d_n & f_n & 0 \\ C & 0 & 0 & 0 \end{array}$$

The table and figure graphically illustrate the possible transitions for a given project at the end of period $n$. Each on-going project will have a separate transition probability matrix. For instance. if project, $P_k$, is in state $WR$ at the start of the assignment period, at the end of the period. based on the events that occurred during that assignment period, the three possible states to move to are $WR$. waiting resource assignment at the start of the new period. $OH$. if some external circumstance put the project on-hold, or $C$. if the project was deemed by the CDCS to be complete.

For a given project, $P_k$ at period $n$, each of the transition probabilities for $P_k$ at the end of period n are defined as:

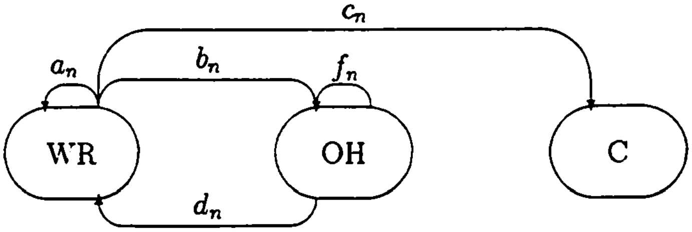

Figure 4.1: State Space Diagram

$a_n$ = $P$[going from $WR$ to $WR$ at the completion of the $nth$ period.]

$b_n$ = $P$[going from $WR$ to $OH$ at the completion of the $nth$ period.]

$c_n$ = $P$[going from $WR$ to $C$ at the completion of the $nth$ period.]

$d_n$ = $P$[going from $OH$ to $WR$ at the completion of the $nth$ period.]

$f_n$ = $P$[going from $OH$ to $OH$ at the completion of the $nth$ period.]

It is important to note that $C$ is an absorbing state as once a project is assumed complete it is removed from consideration. When making resource assignments for a given period, only those projects in state, $WR$, are considered for resource assignment, but the state transition probabilities from $OH$, are necessary for determination of total expected time until completion. Note, that due to the non-stationary behavior of these systems, the transition probability matrix can change with each period.

Based on the above non-stationary probability transition model in conjunction with assistance from Dr. Doug McBeth [1995] the method for determining the expected number of transitions to completion was obtained.

We know that in general for a given transition probability matrix. where $p_{ij}$ is the probability of going from state $i$ to state $j$ at the completion of the period.

$$\sum_{j=0}^{\infty} p_{ij} = 1$$

for an infinite number of states [Wolff 1989]. For our particular model. for a given project $P_k$ at period $n$. we have

$$a_n + b_n + c_n = 1.$$

Let us define.

$$\bar{c}_n = 1 - c_n = a_n + b_n. \tag{4.1}$$

Furthermore. let us assume that each visit to OH is independent of the number of time periods spent in WR and has expected length $\alpha$.

Let. $\tau$ be the remaining time (periods) to get to $C$ if we are on the $nth$ step in WR.

We wish to determine the expected number of remaining steps to get to $C$ if we are in the $nth$ period since project initiation. $X_n$, defined as,

$$X_n = E[\tau_n]$$

We are also interested in finding $X_1$ the total expected time to completion for a project not yet initiated. For each project transition there are three possibilities. transition to completion. transition to on-hold. and continue to require additional resource allocation.

Equation 4.2 for $X_1$ reflects three component parts;

$$X_1 = 1 + a_1 X_2 + b_1[\alpha + X_2] \tag{4.2}$$

the term for the completion of the current period. $a_1X_2$ for the probability of continuing to need resource allocation and expected remaining time to completion. and $b_1[\alpha + X_2]$ to reflect the probability of going on-hold and the expected time during the remaining life of the project for which it will be on-hold. The term $a_k$ is the probability of continuing to require resource allocation. the term $\alpha$ is the expected length of time on-hold each time a project of this type goes on-hold. and $b_k$ is the probability of a project going on-hold at the end of period $k$. The term $\alpha$ is based on an underlying historical distribution obtained from previous project information. Though not considered for this model, if $\alpha$ has large variance then in addition to expected value the entire distribution for $\alpha$ and hence bounds on $X_n$ may need to be considered. In addition. $b_k$ may be fixed. random. or a decreasing function over the life of a project.

Using equation 4.1 we can extend the equation for $X_1$ to obtain an equation for $X_n$.

$$\begin{aligned} X_1 &= 1 + a_1X_2 + b_1[\alpha + X_2] \\ &= 1 + (a_1 + b_1)X_2 + b_1\alpha \\ &= 1 + \bar{c}_1X_2 + b_1\alpha \\ &\vdots \\ X_n &= 1 + \bar{c}_nX_{n+1} + b_n\alpha \end{aligned} \tag{4.3}$$

Since the above equations have a recursive relationship, the solution for $X_n$ can be obtained as follows:

Let

$$\gamma_0 = 1$$

and

$$\gamma_n = \bar{c}_1\bar{c}_2\cdots\bar{c}_n = \prod_{k=1}^{n}\bar{c}_k \tag{4.4}$$

where,

$$\bar{c}_k = a_k + b_k$$

then using equations 4.1, 4.3, and 4.4 we see that for any period $n$,

$$X_n = \frac{\sum_{k=n-1}^{\infty}\gamma_k + \alpha\sum_{k=n}^{\infty} b_k\gamma_{k-1}}{\gamma_{n-1}}.$$

In the special case of $n = 1$ we obtain,

$$X_1 = \sum_{k=0}^{\infty}\gamma_k + \alpha\sum_{k=1}^{\infty} b_k\gamma_{k-1} \tag{4.5}$$

For further model detail and validation see Appendix C.

# CHAPTER 5. RESOURCE ALLOCATION METHODOLOGY

The preceding mathematical model can be extended to make resource allocation decisions on a real-time period-by-period basis.

There are some issues to be considered when prioritizing projects for resource allocation. namely.

- some relationship of expected remaining project duration to available time until due date.
- differing project revenues. and
- resource characteristics and availiability.

Based on this information. the outcome for each assignment period should be two-fold:

- projects ranked (prioritized), and
- resources assigned according to some criteria.

## Alternative Decision Methods

Initially, in developing a resource allocation tool, two questions were asked:

1. Do we need to look just at standard scheduling approaches such as Shortest Processing Time (SPT), First-Come-First-Served (FCFS), etc., [Johnson 1974] in order to determine the best project ranking order, and once ranked, then look separately at how to best assign resources?
2. Or do we need rather to look at all potential project-resource combinations and find the best prioritized ranking of combinations?

Question one was first considered, and four standard tools for ranking projects without considering resources were evaluated as follows:

1. Order projects on a First-Come-First-Served (FCFS) basis [Johnson 1974], ranking the project with the earliest arrival to the CDCS first. Those projects most recently added to the CDCS would come last. The potential pitfall with this method is that it does not consider due date or varying revenue and potentially can result in huge avoidable costs.
2. Order by Shortest Processing Time (STP) or Fewest OPerations Remaining (FOPR) rule [Johnson 1974]: i.e., rank the project nearest completion first. This method could reduce many of the late due dates, but does not really consider a project that may have a tighter schedule. It also does not prioritize higher profit margin projects.
3. Look at the due date for each available project (a Due Date or Slack-type approach) [Johnson 1974]. Determine the total available time remaining until due

date. $D_t$. Calculate $D_t - X_t = S$ (available slack). Order by increasing available slack time: rank the project with the least slack (this may be a negative value) first. This method considers daily penalty costs in addition to the information provided by the previous tools. But it does not consider variable penalty costs (some projects may incur greater per unit penalty costs) and also still does not consider varying revenue levels.

4. Look at a ratio of $X_t/D_t$ and then order by decreasing ratio. This is a variation of other due date type priority rules [Johnson 1974]. The potential pitfalls of this method are the same as the previous ranking method.

## Cost Component

As profitability is the basis for survival for a CDCS, the decision criteria must reflect the revenue and delay cost components, particularly when significant delay penalties or revenue variation exist. As stated previously, the total project revenue is a predetermined amount agreed upon between client and the business responsible for project completion, and can be altered during the project life by mutual agreement due to unforeseen changes in project scope and duration.

Total project cost, $TC_k$, for project, $P_k$, is determined by:

$$TC_k = (\text{periods for completion})\left(\frac{\text{labor costs}}{\text{period}}\right) + \text{ material costs } + \text{ delay costs}$$

We will assume for the moment without loss of generality that because of the labor intensive nature of a CDCS, that the material cost is negligible and will not be considered.

## Labor Factors

In addition to the revenue and delay cost components it is important that the difference between the labor resources be considered. First, we know that certain resources can only do certain tasks. In addition to the critical factors listed previously, a per-period labor cost, $LC$ must be considered. In this model we assume that the ratio of labor cost/work output is fixed, e.g., a more experienced person can cost more, but can also get more accomplished in the same amount of time. As shown in Figure 5.1, Resource A has a lower labor cost but also lower units of work accomplished during the same amount of time. Resources are paid for completing the work content. Thus knowing that the labor cost/work content ratio is fixed, the labor cost component is reduced to a work content consideration. The issue then becomes how best to assign based solely on varying work content.

## Assignment Procedure

For a given assignment period, knowing that revenue and penalty costs will be considered, as well as distinguishing resource characteristics, the decision must be made as to how best to assign all of the limited resources.

We begin by further defining the fraction of work, $U_{R_j}$ resource $R_j$ can accomplish in relation to the average resource during an assignment period. For example, Resource $R_1$ may be less experienced and thus accomplish less than the average amount. Setting $U_{R_1} = .9$, means that on average only 90% of the average work will be accomplished. Equipment resources would typically have a value of $U_{R_j}$ of 1.0.

As profitability is critical to the survival of a CDCS we have chosen as our

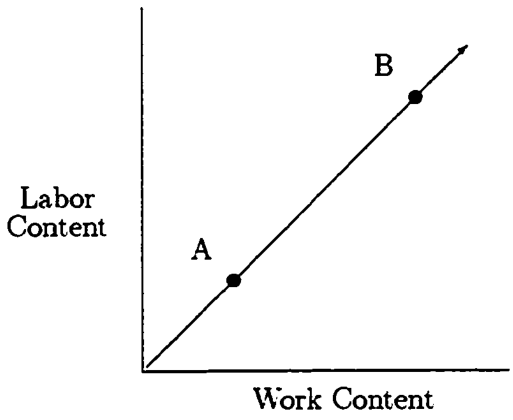

Figure 5.1: Labor Cost versus Work Content

objective for a given assignment period to maximize the profit generated for that period, taking into consideration all the potential resource-project combinations.

Recalling the information available for each project at the start of assignment period, $t$,

1. cumulative project cost
2. remaining task components within a project, $P_k$, $\{T_{jk} : j = 1, \cdots, j\}$
3. transition probabilities, necessary for determining $X_t$, the expected number of remaining time units to get to $C$ (completion) as shown in the model previously, including possible on-hold time. For the remainder of the discussion, we will expand the term, $X_t$, to the term, $X_{t,avg}$ the historical expected value for periods

to $C$ at time $t$. and $X_{t,R_k}$. the expected periods remaining when a project is assigned to a particular resource.

4. total time remaining until deadline (due date) = $D_t$

5. any anticipated per unit time penalty costs anticipated based on $D_t - X_t = S$. when $S < 0$.

We will now define some assignment rules used for decision-making as follows:

- Projects to be added in the future are unknown and will not be considered.

- Projects on-hold will not be considered for resource allocation at the current period. but will be added back in when returning to the $WR$ state.

- At the start of a given assignment period we will consider the feasible resource-project enumerations necessary until each is completed. but not consider the addition of new projects. This is necessary for determination of total time until completion.

- Once a project is assigned to a resource, that resource will remain with it until either:

  - that project is complete. or
  - because of skill limitations another resource must take over.

- Since a resource is going to remain responsible for a project once assigned. it may be necessary to assign more than one project to a given resource and then have that resource alternate between projects as necesary to meet the decision-criteria based prioritization.

- A decision made in the current assignment period is based on the information available in that period, and it is assumed that all current aspects of the CDCS environment will continue for the remainder of the decision time frame. This includes: resource availability, transition probabilities, and resource characteristics. This is a major assumption, but based on the uncertain future, the best information available for the current assignment period. Any changes during the assignment period will be reflected using Bayes' Theorem to adjust information available and hence decisions made in the next assignment period.

- Finally, revenues are generated by a project only when it is actually being worked on, not idle or on-hold, and delay costs are only charged to idle times once the total idle time has reached an extent that will result in a penalty cost. This is valid since projects will not generate revenue until reaching completion, and will only generate penalty costs when they are behind the agreed-upon completion date.

### Decision Procedure

1. At the beginning of the assignment period, for each resource, $R_j$, we can calculate

   $$X_{t,R_j} = X_{t,avg}/U_{R_j},$$

   the expected remaining amount of time it will take resource, $R_j$ to complete project $P_k$.

2. For each project we enumerate all feasible assignment combinations based on the assumptions and resource assignment rules. The worst case would be $P!$.

These resource assignment rules typically reduce the total set of resource-project combinations to resource-constrained trimmed enumeration sets. Each resource will have a limited number of total projects. New projects can be assigned to resources accordingly. When introducing this tool to a CDCS. the initial enumerations can be a trimmed set because certain projects will already be assigned and in progress. In the event of a large combinatorial problem a branch and bound technique could be applied [Hillier 1974].

3. The profit for a given project. $M_t$, generated for the current assignment period is given by

$$M_t = \frac{TR_t - PC}{W_t} \tag{5.1}$$

where.

$TR$ = total revenue upon completion

$PC$ = TDC (total delay cost) − OHC (on-hold days cost)

$W_t = X_t - \alpha \sum_{k=t}^{\infty} b_k \gamma_{k-1} = \sum_{k=t-1}^{\infty} \gamma_k;$

where.

$$TDC = \sum DC$$

and $DC$ is the predetermined delay cost per day delayed.

$W_t$ is the proportion of the total expected remaining time until completion excluding the expected on-hold portion. In other words,

$$W_t = X_t - H_t$$

where, $H_t$ is the expected on-hold portion of time for a project. It is important to exclude the on-hold time from the portion of $X_t$ in the calculation of $M$ so that the calculation only reflects the expected periods for which resources are assigned to the project.

This can be seen in the equation for $X_t$ (which is necessary for determining anticipated time until completion):

$$X_t = \sum_{k=0}^{\infty} \gamma_k + \alpha \sum_{k=1}^{\infty} b_k \gamma_{k-1}.$$

We see the two components of the equation, namely

a working time component; $\sum_{k=0}^{\infty} \gamma_k$
an on-hold component; $\alpha \sum_{k=1}^{\infty} b_k \gamma_{k-1}$.

4. Enumerating all feasible resource-assignment combinations, the combination that maximizes $M_{t,total}$, the sum of $M_t$ for individual projects, will be selected.

At the end of each assignment period all information obtained about projects and resources is used as input for the model during the subsequent assignment period. The question is asked as to how much actual work was accomplished by the resource on the given project and from that the following potentially revised information can be obtained,

- transition probabilities
- and resource factors.

The equation for Bayes' Theorem [Devore 1991, Wolff 1989] useful for distribution modification is given as:

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B)}$$

where

$P(A_i|B)$ is the posterior distribution probability for $A_i$,

$P(A_i)$ is the prior distribution for $A_i$, and

$P(B)$ is the probability of $B$ actually occurring.

Change in project status as well as resource characteristics can be reflected using Bayes' Theorem. It is known that each resource has certain distinguishing characteristics as previously discussed. Therefore, on a period-to-period basis, the additional completion information that is being obtained concerning the resource is useful in establishing an updated proportion of the average amount of work, $U_{R_k}$, that this resource can be expected to accomplish on a given day. For example, as a new resource comes "up to speed" on a particular facet of a job, this can be reflected in the available information concerning the resource using Bayes' Theorem.

Lastly, whenever a CDCS decides to accept a new project, there is some information inherently known about the project. Because of the inherent similarities among projects within a given category, an estimate of transition probabilities and subsequent project duration can be determined. This is based on the historical data that includes:

- expected number of periods each completed project required for completion,
- any on-hold incidences, and if so for how long, and
- resource(s) used.

From this information, an expected value as well as a distribution for the time required for completion can be identified, and a reasonble estimate given to the client for completion when this project is considered in conjunction with other projects currently ongoing and other waiting initiation.

Despite project similarities some differences will be experienced. If, at the end of the first period of work, or at the end of any subsequent period, it is anticipated that the transition probabilities for completion will vary from the expected value, this new information can be reflected using Bayes' Theorem to modify the existing information. The salient feature of this is that there is a reasonable initial estimate for project completion, but as the project progresses it can be refined to reflect the actual characteristics of the particular model.

An example problem using the decision criteria can be found in the last section.

## Observations

As will be illustrated in the example problem of the following section, certain observations seem apparent. The quicker, more flexible resources seem more likely to alternate between projects over the time horizon reflected in the model, while the less flexible resources remain with one project until completion or until they are no longer able to continue due to limitations. This seems reasonable based on information obtained from representative CDCSs.

The idea of assigning resources to projects and remaining responsible for them is also reasonable based on information gathered from sample CDCSs.

The salient feature of using Bayes' Theorem is that it considers the historical data as well as new information in coming up with an expected value and is therefore a gradual refinement of the information.

When looking at the resource-project combinations for a given assignment period. it is possible to use the current information about the resource to find the best assignments. For instance. a project that is behind the anticipated schedule. may be better off being assigned to a resource that has a higher than average accomplishment rate. in order to expedite the project.

Given an environment where a historical basis of information exists or can be established. the use of Bayes' Theorem is beneficial in providing and then refining initial estimates of project duration. as in the types of environments in this research.

# CHAPTER 6. A CLIENT-SERVER CAPACITATED PROBLEM

## Problem Parameters

Consider three projects with varying work content and revenue potential. Let us assume that each project is new and waiting initial start. Table 6.1 represents necessary project information including historical completion time data derived from arbitrary transition probabilities.

Table 6.1: Example Project Information

| $P_k$ | Total Revenue $TR$ | Delay Cost $DC$ | $X_{1,avg}$ | Periods to Due Date, $D$ | Work Content |
|---|---|---|---|---|---|
| $P_1$ | \$180 | \$20 | 6 periods | 8 periods | 3 $T_1$, then 3 $T_2$ |
| $P_2$ | \$175 | \$20 | 5 periods | 6 periods | 3 $T_2$, then 2 $T_1$ |
| $P_3$ | \$150 | \$15 | 4 periods | 4 periods | 4 $T_1$ |

$T_1$ and $T_2$ are the two types of work that the firm can perform. Each project contains either one or both of these task types as shown in Table 6.1.

There are two resources available that will be assigned for this first period with characteristics as shown in Table 6.2. We will look at how to find the best allocatation of resources for the current period given their varying capabilities and current project mix.

Table 6.2: Example Resource Information

| Resource | Task Capabilities | $U_R$ |
|---|---|---|
| $R_1$ | $T_1$ | $0.9(T_1)$ |
| $R_2$ | $T_1$ and $T_2$ | $1.2(T_1)$ $1.0(T_2)$ |

### Mathematical Formulation

We will begin the example problem by an illustration of how the average completion time, $X_{1,avg}$, for task one, $T_{1,avg}$, can be determined based on historical information. We will call it $X_{1,avg,T_1}$.

Note: For this example we have chosen arbitrary values for all variables. We will begin by assuming,

- $\alpha = 1$, and
- $b_k = 0.1 \; \forall \; k$

The following are arbitrary values that could represent historical information collected over time concerning the transition probabilities for this task and given as follows:

$$\begin{aligned} a_1 &= 0.8 \\ a_2 &= 0.6 \\ a_3 &= 0.4 \\ a_4 &= 0.2 \\ a_5 &= 0.001 \\ a_6 &= 0.0001 \end{aligned}$$

Recall that. $a_n + b_n + c_n = 1$

and $\bar{c}_n = 1 - c_n = a_n + b_n$

So. $\bar{c}_n$ can be calculated as follows:

$$\begin{aligned}
\bar{c}_1 &= 0.8 + 0.1 &&= 0.9 \\
\bar{c}_2 &= 0.6 + 0.1 &&= 0.7 \\
\bar{c}_3 &= 0.4 + 0.1 &&= 0.5 \\
\bar{c}_4 &= 0.2 + 0.1 &&= 0.3 \\
\bar{c}_5 &= 0.001 + 0.1 &&= 0.101 \\
\bar{c}_6 &= 0.0001 + 0.1 &&= 0.1001
\end{aligned}$$

So now we also know that $\gamma_0 = 1$ and $\gamma_n = \bar{c}_1\bar{c}_2\cdots\bar{c}_n$.

Therefore $\gamma_n$ can be calculated as follows:

$$\begin{aligned}
\gamma_1 &= 0.9 \\
\gamma_2 &= (0.9)(0.7) &&= 0.63 \\
\gamma_3 &= (0.63)(0.5) &&= 0.315 \\
\gamma_4 &= (0.315)(0.2) &&= 0.063 \\
\gamma_5 &= (0.063)(.101) &&= 0.0063 \\
\gamma_6 &= (0.0063)(0.1001) &&= 0.0063
\end{aligned}$$

Using the above information it is possible to calculate the value for $X_{1,avg,T_1}$, the $T_1$ portion of $P_1$. This is only a portion of the total $X_{1,avg}$ as the project contains both tasks $T_1$ and $T_2$.

So,

$$\begin{aligned} X_{1,avg,T_1} &= \textstyle\sum_{k=1}^{\infty} \gamma_k + \alpha \sum_{k=1}^{\infty} b_k \gamma_{k-1} \\ &= 1 + (.9) + \cdots + (.00063) &&= (1)(.1)(.9 + \cdots + .00063) \\ &= 2.915 + (.1)(1.915) &&= 3.106 \text{ time periods} \end{aligned}$$

If we knew that everything remained the same for the life of the project we could continue to calculate $X_{2,avg,T_1}$, $X_{3,avg,T_1}$, $\cdots$, for this project. However, we must consider that this value for $X_{2,avg,T_1}$ would only be valid if at the end of the first period the state transition is from $WR$ to $WR$ and work has been accomplished exactly as expected. Any changes in the factors that affect $P_1$ would require a revised transition matrix $E_2$ to deterine $X_{2,avg}$ using Bayes' Theorem prior to making a decision for the next period.

Our objective: choose the set of resource-project pairs that maximize the profit for a given time period as given in Equation 5.1 in the section on decision criteria. Only available projects are considered in the decision for today, as we cannot know what projects will be added at a later date.

## Initiation

At the start of each project we can show graphically what each project content includes (see Figure 6.1), where $S$ = slack, and $DD$ = due date. Note that the values of $X_{1,avg}$ for each project have been arbitrarily set to integer values for this initial day example.

Following are the details of the example resource asignment until one of the projects reaches completion.

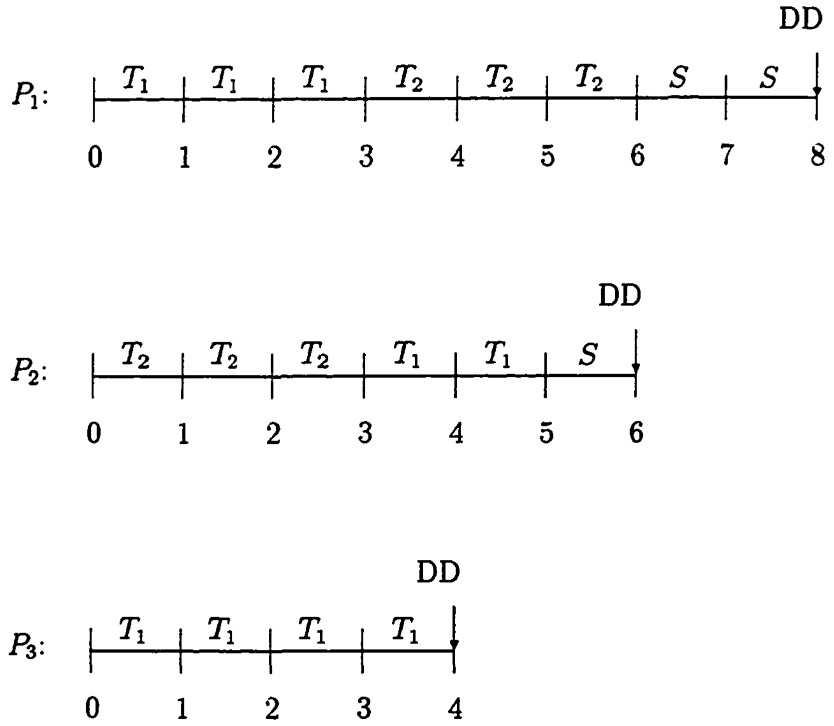

Figure 6.1: Example Project Duration Time Line

### Period One

We can enumerate the possible resource-assignment combinations for Period One as shown in Table 6.3. Now recall that for each resource, $R$, it is given that

$$X_{t,R} = X_{t,avg}/U_R,$$

and the amount of profit per project per time period is:

$$M_t = \frac{TR_t - PC}{W'_t}$$

where.

$$W'_t = X_t - a \sum_{k=t}^{\infty} b_k \gamma_{k-1} = \sum_{k=t-1}^{\infty} \gamma_k. \tag{6.1}$$

Table 6.3: Period One Resource-Assignment Options

| Option | $P_1$ | $P_2$ | $P_3$ |
|---|---|---|---|
| 1 | $R_1$ | $R_2$ | none |
| 2 | $R_1$ | none | $R_2$ |
| 3 | $R_2$ | none | $R_1$ |
| 4 | none | $R_2$ | $R_1$ |

### Option One

Figure 6.2 illustrates the assignment of resources for Option One.

$P_1$: Because of limitations. $R_1$ can only do $T_1$ so it will be able to work on the project for only the first three time periods. $U_{R_1} = 0.9$ so the $X_{1,R_1,T_1}$ portion would be expected to take $3\frac{1}{3}$ time units. The remaining three periods as shown in Table 6.1 would then be assigned to a resource, $R_2$, when that resource has completed project $P_2$ and the expected total time for $P_1$ would be $6\frac{1}{3}$ time periods.

$P_2$: $R_2$ can do both tasks so it can remain with project for entire time. $U_{R_2} = 1.2$ for $T_1$ and 1.0 for $T_2$. so the entire project is expected to take:

$$\frac{3}{1.2} + \frac{2}{1.0} = 2.5 + 2 = 4.5 \text{ time periods.}$$

$P_3$: This cannot begin until $R_1$ or $R_2$ is available and hence will use $R_1$ and the historical expected completion duration of 4 periods as shown in Table 6.1.

Therefore based on equation 6.1:

$$W^{*}_{1.P_1} = 5.7$$

where. 2.9 is the $W^{*}_1$ component for task one and 2.8 is $W^{*}_1$ for task two.

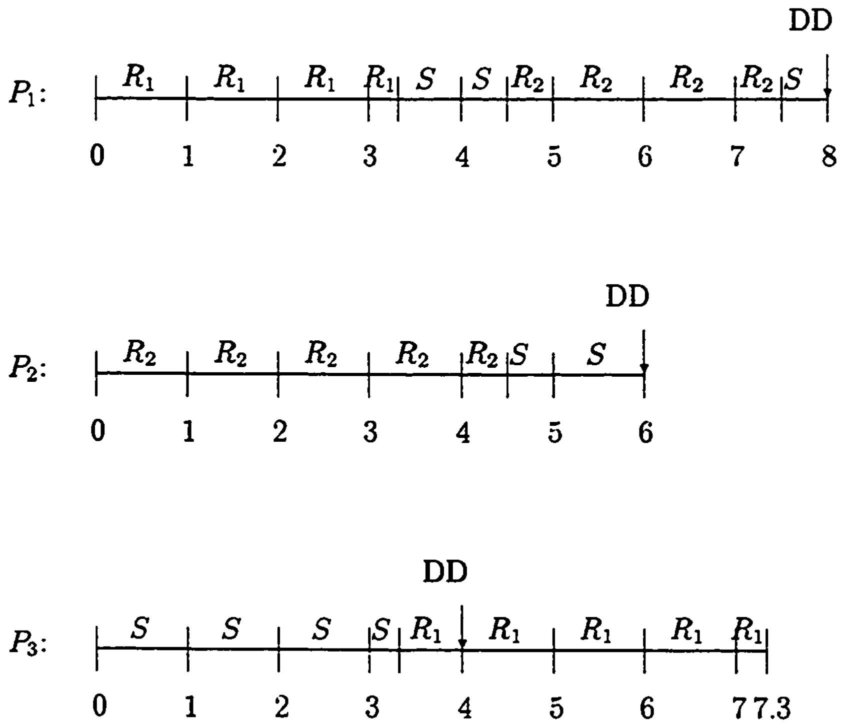

Figure 6.2: Option One Project Completion Time Line

Using resource assignment Option One we calculate.

$$M_{1,P_1} = \frac{TR_1 - PC}{W_t} = \frac{180-0}{5.7} = \$31.56$$

where $M_{1,P_1}$ is the profit generated this period for project $P_1$ and will be added to the profit for $P_2$ and $P_3$ to get total profit for the period, $M_{1,total}$.

Current period profits for other projects are:

$$M_{1,P_2} = \$42.68$$

$$M_{1,P_3} = \$27.04,$$

giving a total profit of

$$M_{1,total} = \$101.28.$$

## Option Two

The resource-project pairings for option two were given in Table 6.3. An illustration of Option Two is also portrayed graphically in Figure 6.3.

The profit generated today if option two is chosen is

$$\begin{aligned}
P_1:\ M_{1,P_1} &= \tfrac{180-(1.33)(20)}{5.7} &&= \$26.91,\\
P_2:\ M_{1,P_2} &= \tfrac{175-(2.33)(20)}{4.6} &&= \$27.91,\\
P_3:\ M_{1,P_3} &= \tfrac{150-0}{2.9} &&= \$51.72,\\
\text{and}\ M_{1,total} &= \$106.54.
\end{aligned}$$

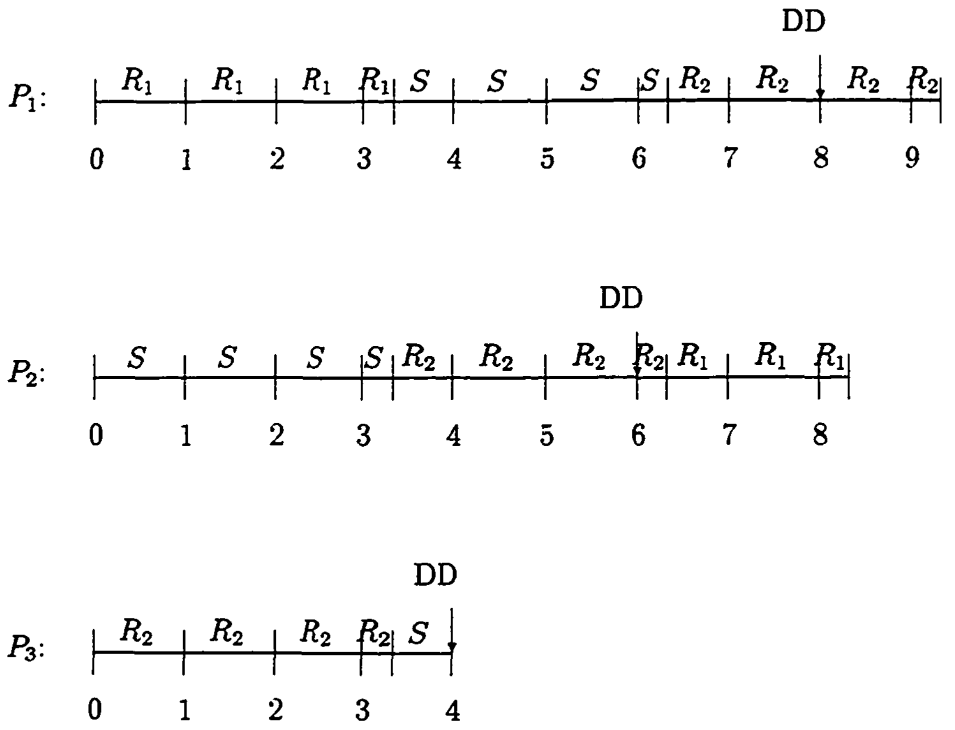

Figure 6.3: Option Two Project Completion Time Line

## Option Three

Continuing with the same technique as the previous two options, eliminating the graphical illustration, the profits are

$$\begin{array}{llcccl}
P_1: & M_{1,P_1} & = & \frac{180-0}{5.3} & = & \$33.96, \\
P_2: & M_{1,P_2} & = & \frac{175-(4.5)(20)}{4.8} & = & \$17.71, \\
P_3: & M_{1,P_3} & = & \frac{150-(.44)(15)}{4.3} & = & \$33.35,
\end{array}$$

and $M_{1,total} = \$85.02$.

## Option Four

The procedure is repeated with Option Four giving

$$\begin{aligned} P_1:&\quad M_{1.P_1} &=& \tfrac{180-(2.44)(20)}{5.8} &=& \$22.62, \\ P_2:&\quad M_{1.P_2} &=& \tfrac{175-0}{4.5} &=& \$38.89, \\ P_3:&\quad M_{1.P_3} &=& \tfrac{150-(.44)(15)}{4.3} &=& \$33.35, \\ \text{and}&\quad M_{1,total} &=& \$94.86. \end{aligned}$$

## Period One Overview

Information regarding the four options for Period One is described in Table 6.4.

Table 6.4: Period One Overview

| Option | $M_1$ | Total Delay Periods | Projects Delayed |
|---|---|---|---|
| 1 | \$101.28 | 3.3 | $P_3$ |
| 2 | \$106.54 | 3.67 | $P_1, P_2$ |
| 3 | \$85.02 | 4.94 | $P_2$ |
| 4 | \$94.86 | 2.88 | $P_1$ |

Based on the profit maximization criteria the assignment decision for Period One is shown in Table 6.5.

At the end of Period One a review is made of the work that has been accomplished and based on work completed, the transition probabilities for project one and three are altered as necessary using Bayes' Theorem to reflect the new information.

The new information resulted in the following values for $X_{2,avg}$ for the projects, which will used for decision-making for the second assignment period.

Table 6.5: Period One Resource Assignments

| $P_k$ | $R_j$ |
|---|---|
| 1 | 1 |
| 2 | none |
| 3 | 2 |

$P_1$: $X_{2,avg}$ = 5 time periods, 2 for $T_1$ and 3 for $T_2$

$P_2$: $X_{2,avg}$ = 5 time periods

$P_3$: $X_{2,avg}$ = 2 time periods

Also, based on the assumption that once a resource begins a project it will remain with it until either:

- the project is finished or goes on hold, or
- the resource is no longer capable of performing the required work

we know that $P_1$ is now the responsibility of $R_1$ and $P_3$ is the responsibility of $R_2$.

### Period Two

We know that $P_1$ has been assigned to $R_1$ and $P_3$ to $R_2$. We also know that because of capability limitations, $R_1$ cannot work on $P_2$, so the only options available for consideration at the start of the second period are shown in Table 6.6.

The profit calculations for each option gives

Table 6.6: Period Two Resource-Assignment Options

| Option | $P_k$ | $R_j$ |
|---|---|---|
| 1 | 1 | 1 |
| | 2 | none |
| | 3 | 2 |
| 2 | 1 | 1 |
| | 2 | 2 |
| | 3 | none |

$$\begin{aligned} \text{Option One: } M_2 &= \$111.36 \\ \text{and, Option Two: } M_2 &= \$116.97. \end{aligned}$$

For Period Two the decision is made to follow Option Two,

- let $R_1$ continue with $P_1$, and
- let $R_2$ set $P_3$ aside for the day and switch to working on $P_2$.

At the end of the the second period an evaluation of work completed is made and the transition matrix updated. The completion times are calculated as before giving,

$$\begin{aligned} P_1\text{: } X_{3,avg} &= 1(T_1) + 3(T_2) && = 4 \text{ time periods,} \\ P_2\text{: } X_{3,avg} &= 1.5(T_2) + 2(T_1) && = 3.5 \text{ time periods,} \\ \text{and, } P_3\text{: } X_{3,avg} &= 2(T_2) \text{ time periods.} \end{aligned}$$

## Period Three

So now at the start of Period Three there are again two options to consider as shown in Table 6.7.

Table 6.7: Period Three Resource-Assignment Options

| Option | $P_k$ | $R_j$ |
|---|---|---|
| 1 | 1 | 1 |
|  | 2 | none |
|  | 3 | 2 |
| 2 | 1 | 1 |
|  | 2 | 2 |
|  | 3 | none |

The profit calculations for each options result in

$$\begin{aligned} \text{Option 1: } M_3 &= \$124.22, \\ \text{and, Option 2: } M_3 &= \$120.24. \end{aligned}$$

For Period Three the decision is made to follow Option One,

- let $R_1$ continue with $P_1$, and
- let $R_2$ set $P_2$ aside for the day and switch to working on $P_3$.

At the end of the assignment period an evualation of work completed is made and the transition matrix is updated. The completion times are calculated as before giving

$$\begin{aligned} P_1\text{: } X_{4,avg} &= 0.3(T_1) + 3(T_2) && = 3.5 \text{ time periods}, \\ P_2\text{: } X_{4,avg} &= 1.5(T_2) + 2(T_1) && = 3.5 \text{ time periods}, \\ \text{and, } P_3\text{: } X_{4,avg} &= 0.5(T_2) \text{ time periods}. \end{aligned}$$

### Period Four

At the start of the fourth period there are again two options to consider as shown in Table 6.8.

Table 6.8: Period Four Resource-Assignment Options

| Option | $P_k$ | $R_j$ |
|---|---|---|
| 1 | 1 | 1 |
| | 2 | none |
| | 3 | 2 |
| 2 | 1 | 1 |
| | 2 | 2 |
| | 3 | none |

The profit calculations for each option gives

$$\begin{aligned} \text{Option 1: } M_4 &= \$131.44, \\ \text{and, Option 2: } M_4 &= \$130.18. \end{aligned}$$

For Period Four the decision is made to follow Option 1,

- let $R_1$ continue with $P_1$ and complete all possible in that project
- and let $R_2$ set $P_2$ aside until work is completed on $P_3$.

## Observations

Observations after Period Four:

- Project Three is now complete. The overall project completion diagram is shown in Figure 6.4.
- The quicker, more flexible resource, $R_2$, alternated between projects while the less flexible resource stuck with the one started which seems logical.
- Though not considered in the example, it is possible for three other scenarios to have occurred:

- one of the projects could have gone on hold
- a project could have been removed entirely from consideration (scrapped)
- additional project(s) could have been added to consideration thus possibly changing the decision.

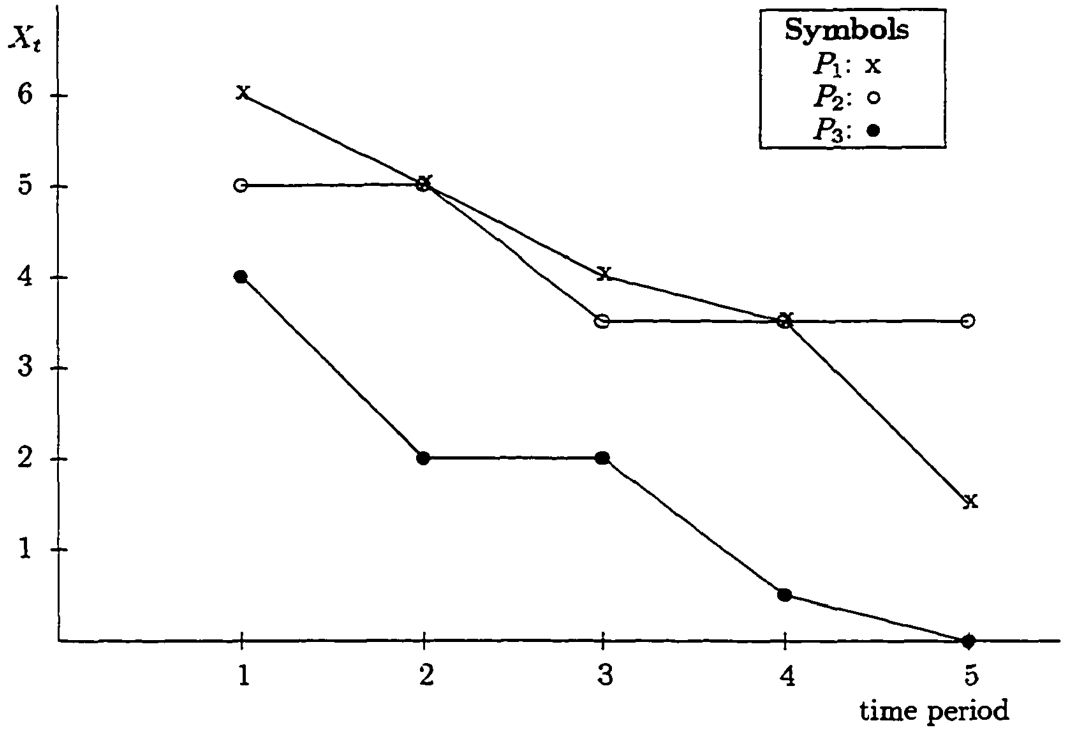

Figure 6.4: Project Completion Rate Diagram

# CHAPTER 7. CONCLUSION

Research began with the categorization and determination of the distinguishing characteristics of a client-driven capacitated system (CDCS). From there the issue of how to best allocate the limited resources of these sytems was considered. Each resource allocation period consists of a cycle of decision-making at the beginning of the period and results at the end of the period. A revenue based decision-criteria was developed and the decision procedure was described. The research concluded with an illustrative example of the model application.

## Contributions

The contributions of this research fall into two categories:

1. The first significant component of this research is the parameterization of the CDCS, as this will be invaluable for future analysis of these types of systems in other aspects than the resource allocation (scheduling) aspect basis of this research.

2. The second contribution has been the extension of the CDCS characterization into a revenue-based resource allocation tool for use in these systems. The Bayesian historical basis and model refinement component is an additional contribution to the model.

## Strengths and Limitations

There are a number of significant strengths of this research model:

- The number of resources can be dynamic from period to period.
- New projects can be added. existing projects put on-hold. and completed projects can be deleted from the model.
- Using transition probabilities. the expected project completion duration can be determined.
- As a project progresses, the prior distribution can be refined using Bayes' Theorem and information reflecting actual events.
- As resource characteristics change over time. these can also be modified using Bayes' Theorem.

There are also a number of limitations and assumptions that have been made in using this model. These have been addressed as appropriate throughout the model development. In the following section on future extensions. many of the assumptions are addressed as candidates for modification or extension.

# CHAPTER 8. FUTURE APPLICATIONS AND EXTENSIONS

Consideration can be given to the following areas.

1. A project in progress remains with the resource assigned in the previous period and therefore only those resources needing new assignments will be considered in the decision model.
2. A flexible resource has a set of its own projects. choosing the "best" for him(her)self and then leaving the other resources the remainder of the projects.
3. Look at sensitivity of $X_t$.

   Robustness:

   - revenue disparity
   - delay disparity
   - project slack time
4. Consider different ways of looking at revenue.
5. It may be possible to generalize the quantitative model to more than three states as in this model and still get a solution for $X_1, \cdots, X_n$ in terms of matrices as shown in the chapter on quantitative model development.

6. Consider the limitations on model size for enumeration possibilities and alternative approaches in the event of a large combinatorial problem.
7. Incorporate into the model the cost of materials and overhead instead of only the cost of labor.
8. Consider alternative client prioritization options.
9. Incorporate the effect of bounds on $X_t$, as opposed to simply considering the expected value of the distribution.

## REFERENCES

[Al-Bahar 1990] Al-Bahar. J. F. and Crandall. K. C.. "Systematic Risk Management Approach for Construction Projects", *Journal of Construction Engineering and Management*. vol. 116. September 1990. pp. 533-547.

[Arditi 1989] Arditi. D. and Patel. B. K.. "Impact Analysis of Owner-Directed Acceleration". *Journal of Construction Engineering and Management*. vol. 115. 1989. pp. 144-157.

[Bishop 1995] Bishop Engineering, data from personal interview. September 1995.

[Cooper 1993] Cooper, Kenneth G.. "The Rework Cycle". *EMR*. Fall 1993. pp. 4-12.

[DeHamer 1995] DeHamer. Rick. A.P., P.C., data from personal interview, September 1995.

[Devore 1991] Devore J. L., *Probability and Statistics for Engineering and Sciences*. Wadsworth, Inc.. Belmont. CA, 1991.

[French 1995] French, Simon. "Uncertainty and Imprecision: Modeling and Analysis". *Journal of the Operational Research Society*. vol. 46, 1995, pp. 70-79.

[Guinet 1991] Guinet. A.. "Textile Production Systems: a Succession of Non-Identical Parallel Processor Systems". *Journal of the Operational Research Society.* vol. 42. no. 8. 1991. pp. 655-671.

[Handfield 1994] Handfield. Robert B.. "Effects of Concurrent Engineering on Make-to-Order Products". *IEEE Transaction on Engineering Management.* vol. 41. no. 4. 1994, pp. 384-393.

[Hillier 1974] Hillier. Frederick S., and Lieberman. Gerald J.. *Operations Research.* Holden-Day. San Francisco. 1974.

[Johnson 1974] Johnson, Lynwood. A. and Montgomery. Douglas C.. *Operations Research in Production Planning, Scheduling, and Inventory Control.* John Wiley and Sons. New York, 1974.

[Laufer 1990] Laufer. A. and Cohenca, D. "Factors Affecting Construction-Planning Outcomes," *Journal of Construction Engineering and Management.* vol. 116, 1990, pp. 135-156.

[Martin 1975] Martin. J. J., *Bayesian Decision Problems and Markov Chains.* Robert Kreiger Publishing Company, Huntington. NY. 1975.

[McBeth 1995] McBeth, Douglas W., personal communication, November, 1995.

[Morgan 1966] Morgan. B. W., *Introduction to Bayesian Statistical Decision Processes*, Prentice-Hall. Inc., Englewood Cliffs. NJ. 1966.

[Muth 1963] Muth. J.F. and Thompson, G.L.. *Industrial Scheduling.* Prentice-Hall. Inc., Englewood Cliffs, NJ, 1966.

[Randhawa 1995] Randhawa, S. U. and Smith, T.A., "An Experimental Investigation of Scheduling Non-Identical, Parallel Processors with Sequence-Dependent Set-up times and Due-Dates", *International Journal of Productivity Research*, vol. 33, no. 1, 1995, pp. 59-69.

[Schei 1990] Schei, K. G., "Small Project Management", *ASCE Journal of Civil Engineering*, vol. 60, 1990, pp. 42-44.

[Smith 1988] Smith, J.Q., *Decision Analysis, A Bayesian Approach*, Chapman and Hall, Ltd., London, 1988.

[Tuttle 1994] Tuttle, Quentin C., *Implementing Concurrent Project Management*, PRT Prentice Hall, Englewoods Cliffs, NJ, 1994.

[Webb 1995] Webb, Charlie, Allstate Insurance, data from personal interview, September, 1995.

[Wolff 1989] Ronald W. Wolff, *Stochastic Modeling and the Theory of Queues*, Prentice-Hall, NJ, 1989.

[Zhang 1995] Zhang, H.C. and Huang, S.H. "Applications in Neural Networks in Manufacturing: a State-of-the-Art Survey", *International Journal of Productivity Research*, vol. 33, no. 3, pp. 705-728.

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In closing, I wish to thank God for the talents He has given me and the terrific family and friends He has blessed me with that have been my encouragers through this all.

# APPENDIX A. INTERVIEW QUESTIONNAIRE

The following is the list of questions used in interviewing the three representative firms for model development.

## Project Characteristics

1. List the categories of projects your firms works on.
2. Which of those do you personally work on?
3. What other activities compete for time during your work day, and in what proportion?
4. How may projects are typically ongoing (yourself and total firm)?
5. Can these projects easily be subdivided into smaller tasks? What is the overall structure of typical projects?
6. What similarities are there among project as far as complexity?
7. What past sources of information (references, databases) or experiences do you use to help in completion of current projects?
8. Do you know with certainty at the start of the project what task order and project content will be? If not, why not?

## Resource Characteristics

1. Do you work alone of in conjunction with others to complete projects?
2. If with others, how is the work allocated? How is it determined who will work on a project (selection process)?
3. What special skills and\or experience is needed?
4. Can certain tasks be done by only certain people?
5. How quickly can someone become proficient (task specific)?

## Client Characteristics

1. Are clients prioritized, and if so, how?
2. Are there any financial or nonfinancial considerations in deciding whose work to prioritize?
3. How does demand level vary between clients and how is that considered in decision-making?
4. Do you keep records of clients/jobs?
5. Do you ever turn down a project because you are overloaded? What then happens to that project?

## Completion Characteristics

1. Is quality a significant factor in project completion?

2. Do you have a good estimate of how long it should take you to complete each project? How do you make a "good" estimate? How do you know it is a "good" estimate?

3. Does each project have a predetermined completion date? If so, how is it determined?

4. Are there review points or accountability points during the course of a project? If so, is the client involved?

5. Is intermittent work on the project feasible? If so, when you return to work on the project is there any duplication of effort (analogous to set-up time in manufacturing)?

6. Are there multiple paths to successful completion? Can there be more than one way?

7. Can certain decisions result in dead-ends, redundancy, or undetected errors (an iterative nature)?

8. How do you decide which project to work on at the start of the day?

9. Do you typically get your projects done on time? If not, what percentage is overdue?

10. What do you do if you are behind an anticipated deadline?

11. Are there ways to expedite a project?

12. How do you know when a project is satisfactorily completed?

13. Do you ever discover problem later after apparent completion (days. weeks. or months later)? If so, what is done about it?

# APPENDIX B. GENERIC CHARACTERISTICS

## Project Characteristics

### Similarities

- well-defined categories of projects
- multiple types of categories of projects
- specific types of responsibilities handled by interviewee and other tasks that can be delegated to others
- interruptions common, particularly the phone, but client contact is viewed as critical
- multiple projects in progress or in need of attention at the same time
- varying complexity and durations of projects within a business
- useful sources of information are available for decision-making
- dependence on information from external sources (the client or third party)

### Differences

- ease of subdividing projects into tasks (this was both within a business and among businesses)
- ability to predetermine task order and project content

## Resource Characteristics

### Similarities

- work in conjunction with others to complete projects
- specific skills or training (expertise) required
- only certain people can do certain tasks

### Differences

- method of task allocation
- time to achieve proficiency

## Client Characteristics

### Similarities

- clients are prioritized
- varying demand level among clients
- client demand level impacts decision-making

- records are maintained for each project 'and/or' client
- projects or clients are turned down on occasion
- clients will typically go to a competitor when rejected

### Differences

- reasons for prioritization
- reasons for turning down a client

## APPENDIX C. COMPUTATIONAL CONSIDERATIONS FOR $X_n$

Recall from previous notation that:

$$\begin{aligned} a_n &= P[\text{going from } WR \text{ to } WR \text{ on the } nth \text{ step in } WR] \\ b_n &= P[\text{going from } WR \text{ to } OH \text{ on the } nth \text{ step in } WR] \\ c_n &= P[\text{going from } WR \text{ to } C \text{ on the } nth \text{ step in } WR]. \end{aligned}$$

Also,

$$X_n = \frac{\sum_{k=n-1}^{\infty} \gamma_k + \alpha \sum_{k=n}^{\infty} b_k \gamma_{k-1}}{\gamma_{n-1}} \tag{C.1}$$

where

$$\begin{aligned} \gamma_0 &= 1, \text{ and} \\ \gamma_n &= \bar{c}_1 \bar{c}_2 \cdots \bar{c}_n = \textstyle\prod_{k=1}^{n} \bar{c}_k \end{aligned}$$

and, $\alpha$ is the expected length of time on hold each time it is on hold, and, $b_k$ is the probability of going on hold.

### Computing $X_n$

We begin by looking at the behavior of the first term of $X_n$. By fixing the probabilities such that

$$\begin{aligned} c_n &= q, \text{ and} \\ 1 - c_n &= \bar{c}_n = p, \end{aligned}$$

then from the definition of $\gamma_n$,

$$\begin{aligned}
\gamma_1 &= p \\
\gamma_2 &= p^2 \\
&\vdots \\
\gamma_k &= p^k.
\end{aligned}$$

The first term for $X_1$ using Equation C.1 is given by:

$$\sum_{k=0}^{\infty} p^k.$$

We recognize this as a power series which converges to

$$\frac{1}{1-p}.$$

If we break this term apart and perform a finite number of computations we get

$$\sum_{k=0}^{n} p^k = \frac{1-p^{n+1}}{1-p}$$

and an error term of

$$\sum_{k=n+1}^{\infty} p^k = \frac{p^{n+1}}{1-p}.$$

If given an error bound of $\epsilon$ near zero, we can use a numerical example to illustrate that the error term quickly goes to the bound $\epsilon$.

Let. $c_n = .6$, $\bar{c}_n = p = .4$, and $\epsilon = 10^{-3}$.

The $\gamma_i$ terms would be $\gamma_0 = 1, \gamma_1 = .4, \gamma_2 = .4^2, \gamma_3 = .4^3 \cdots \gamma_n = .4^n$.

For $n = 1$, the first term is

$$\begin{aligned}
\textstyle\sum_{k=n-1}^{\infty} \gamma^k &= \textstyle\sum_{k=0}^{1} p^1 + \sum_{k=2}^{\infty} p^1 \\
&= \tfrac{1-p^{1+1}}{1-p} + \tfrac{p^{1+1}}{1-p} \\
&= \tfrac{1-.4^2}{1-.4} + \tfrac{.4^2}{1-.4} \\
&= 1.4 + .267.
\end{aligned}$$

It follows that for increasing values of $n$

$$\begin{array}{lll}
n=2: & \frac{1-.4^3}{1-.4}+\frac{.4^3}{.6} & =1.56+.1067 \\
n=3: & \frac{1-.4^4}{1-.4}+\frac{.4^4}{.6} & =1.624+.043 \\
n=4: & 1.650+.017 & \\
n=5: & 1.660+.007 & \\
n=6: & 1.664+.003 & \\
n=7: & 1.6656+.001 & \\
n=8: & 1.6662+.0004 & \\
 & \vdots & \\
n=12: & 1.66666+.00001 & \\
n=13: & 1.66666+5\times 10^{-6} &
\end{array}$$

For a constant $p$ it is easy to see that the error term rapidly approaches $\epsilon$ for even a small $n$. The rate of error term decrease, though, is dependent on the value of $p$. The second term, $\alpha\sum_{k=0}^{\infty} b_k\gamma_{k-1}$ would behave in a similar manner. So, although Equation C.1 has terms containing infinite sums, the model is still useful for scenarios containing finite numbers of periods.

### Typical Scenario with $p$ as a Decreasing Function

In the previous error term discussion we arbitrarily set $p_n$ to a constant. For this scenario we now set $p_n = \bar{c}_n = a_n + b_n$ to be a decreasing function which is as it should be since a project will over time increase its probability of reaching completion. We will show that for this case, the expected time remaining, $X_n$, for a project, will converge to the duration of the remaining period.

Chosen arbitrarily for this example, $\alpha = 1$ and values for $a_n$ and $b_n$ are as shown

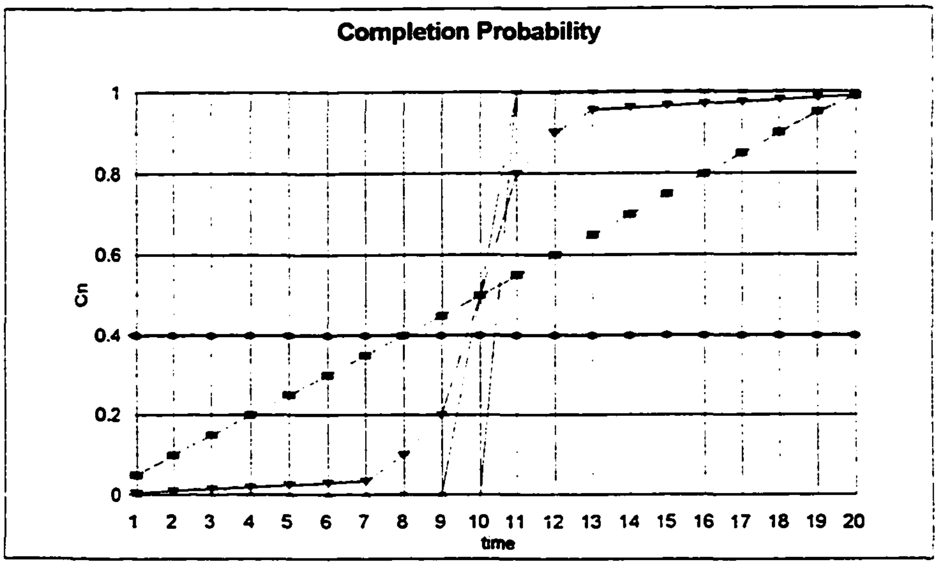
Table C.1: Typical Scenario Results

| Period | $a_n$ | $b_n$ | $c_n$ | $p_n$ | $\gamma_n$ | $X_n$ | Total Time |
|---|---|---|---|---|---|---|---|
| 1 | .949 | .05 | .001 | .999 | .999 | 4.6544 | 4.6544 |
| 2 | .945 | .05 | .005 | .995 | .994 | 3.6080 | 4.6080 |
| 3 | .935 | .04 | .025 | .975 | .969 | 2.5709 | 4.5709 |
| 4 | .46 | .04 | .5 | .5 | .4846 | 1.5702 | 4.5702 |
| 5 | .04 | .01 | .95 | .05 | .02423 | 1.0603 | 5.0603 |
| 6 | .004 | .001 | .995 | .005 | .00012 | 1.0060 | 6.0060 |
| 7 | .0005 | .0005 | .999 | .001 | 1.21E-7 | 1.0015 | 7.0015 |
| 8 | 5E-05 | 5E-05 | .9999 | .00001 | 1.2E-11 | 1.0002 | 8.0002 |

in Table C.1.

Results in Table C.1 show that for the first four periods the total project time progresses toward 4.57. The slight decrease in these values reflects the decrease in the $\gamma$ values. Though there is slight variation in the first four values, it is important to look at the values in light of a scheduling tool and note that for a CDCS this method still provides a good estimate of expected time to completion. Also note that after Period Five has been reached which is the period during which work was expected to reach completion the behavior logically changes. Beginning with Period Five the values for remaining time until completion rapidly decrease toward $X_n = 1$ as shown in Table C.1. This behavior makes sense in that the expected amount of time for completion will reach only the current period as the expected probability of continued work, $a_n$, and going on hold, $b_n$, go toward zero.

An illustration of behavior of $X_n$ for different $c_n$ scenarios is shown in Figure C.1. Figure C.1a shows the completion probability function for various $c_n$ and Figure C.1b shows the corresponding expected duration.



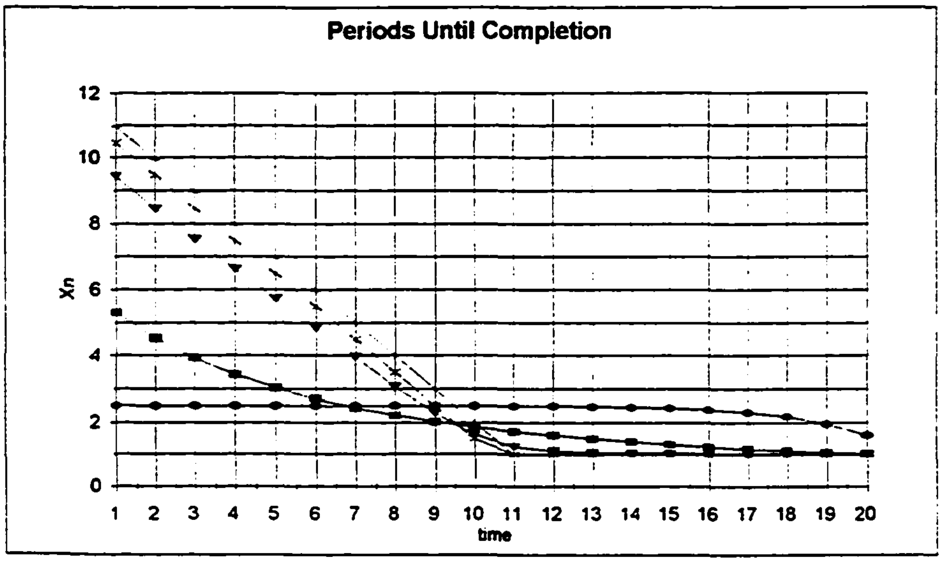


Figure C.1: Expected Completion Duration versus Completion Probability